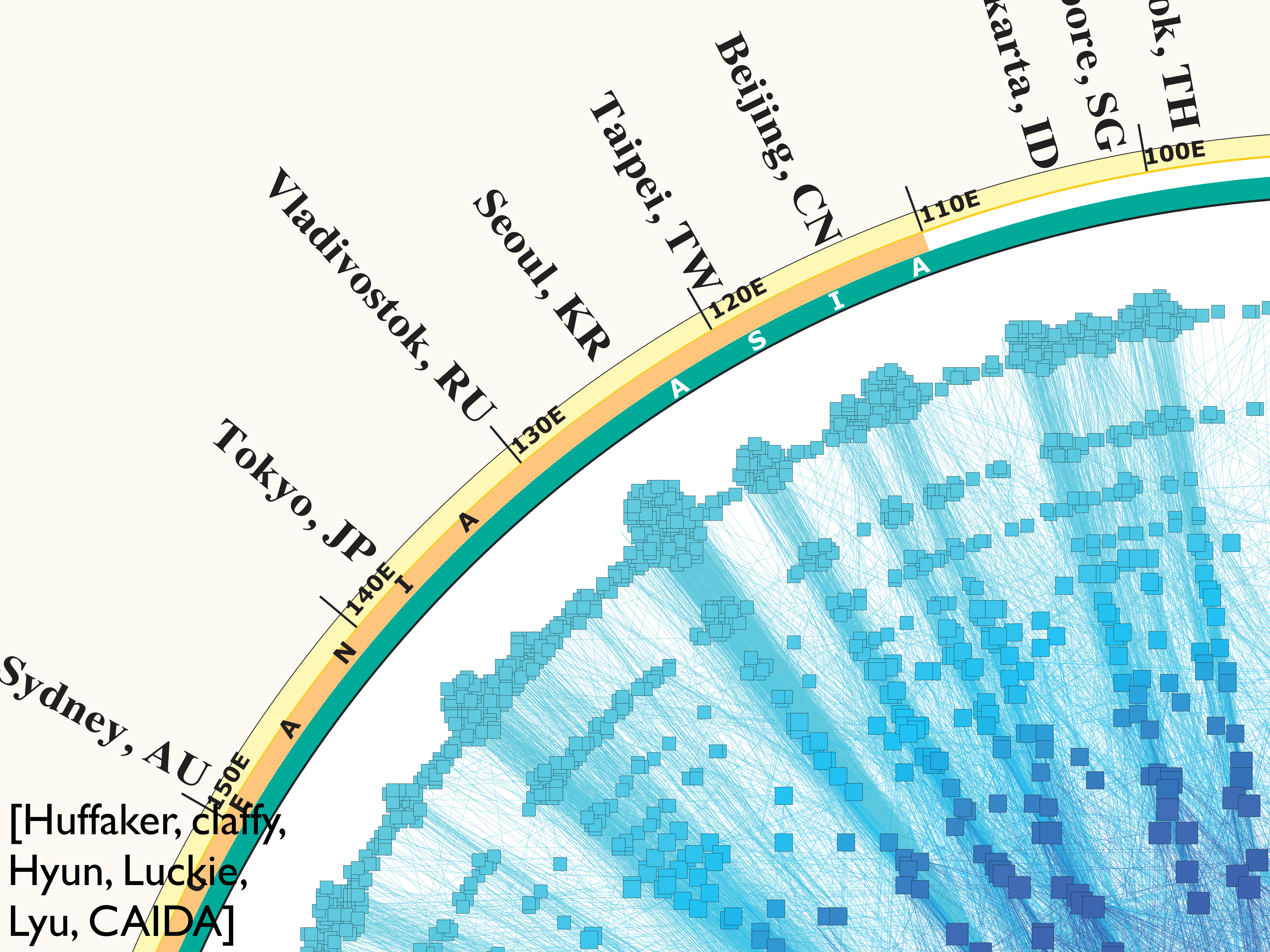
Networks and Games

Brighten Godfrey

Discover Engineering CS Camp

July 24, 2012

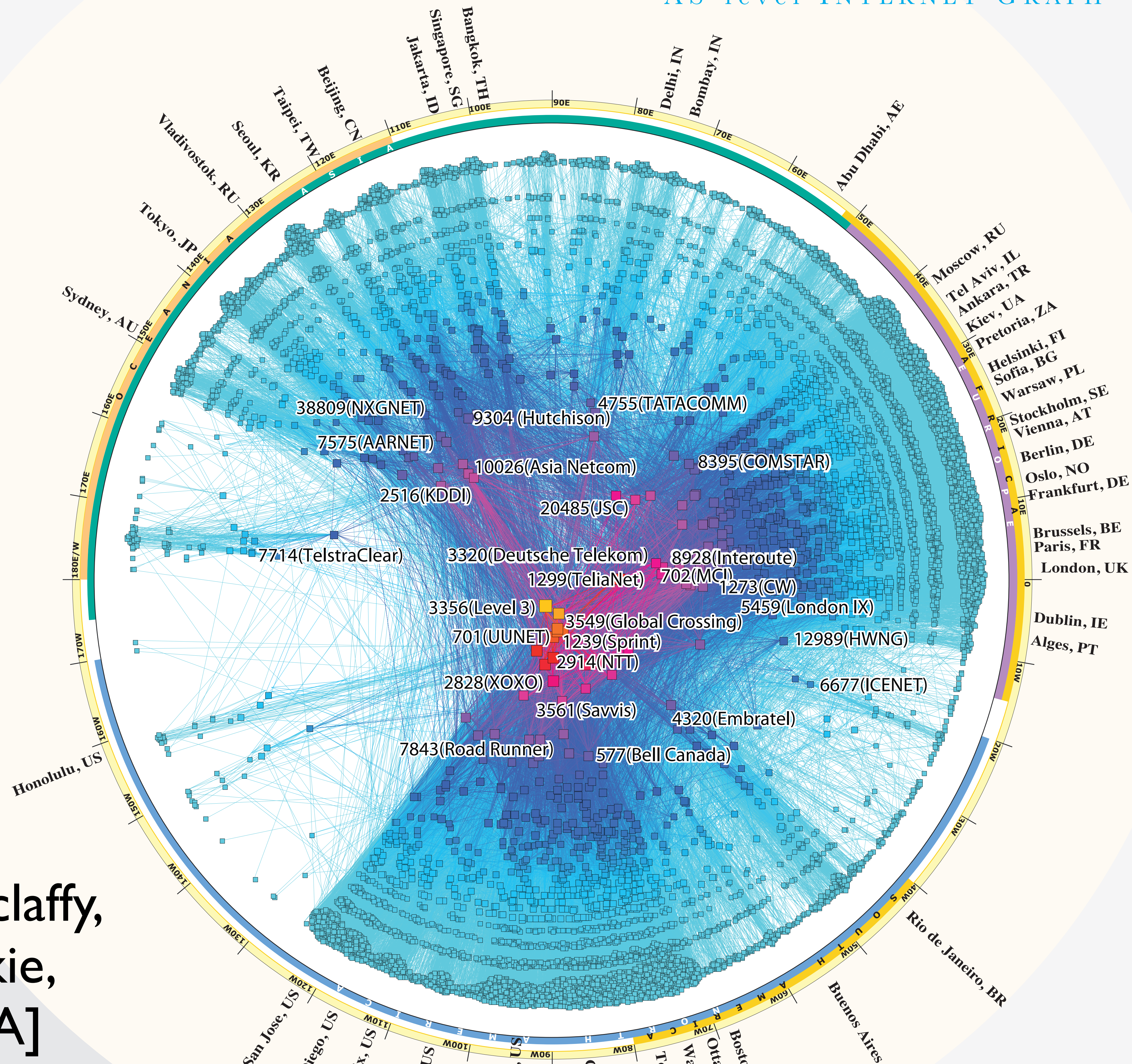
Demo



[Huffaker, claffy, Hyun, Luckie, Lyu, CAIDA]

IPv4 & IPv6 INTERNET TOPOLOGY MAP JANUARY 2009

AS-level INTERNET GRAPH



[Huffaker, claffy, Hyun, Luckie, Lyu, CAIDA]

Games & networks: a natural fit

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Game theory

Studies interaction between selfish agents

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Studies interaction between selfish agents

Networking

Enables interaction between agents

Games & networks: a natural fit

Game theory
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Networking
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Networks make games happen!

Game theory basics

Game theory

Two or more players

For each player, a set of strategies

For each combination of played strategies, a payoff or utility for each player

Game theory

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Blue player strategies

Red player strategies

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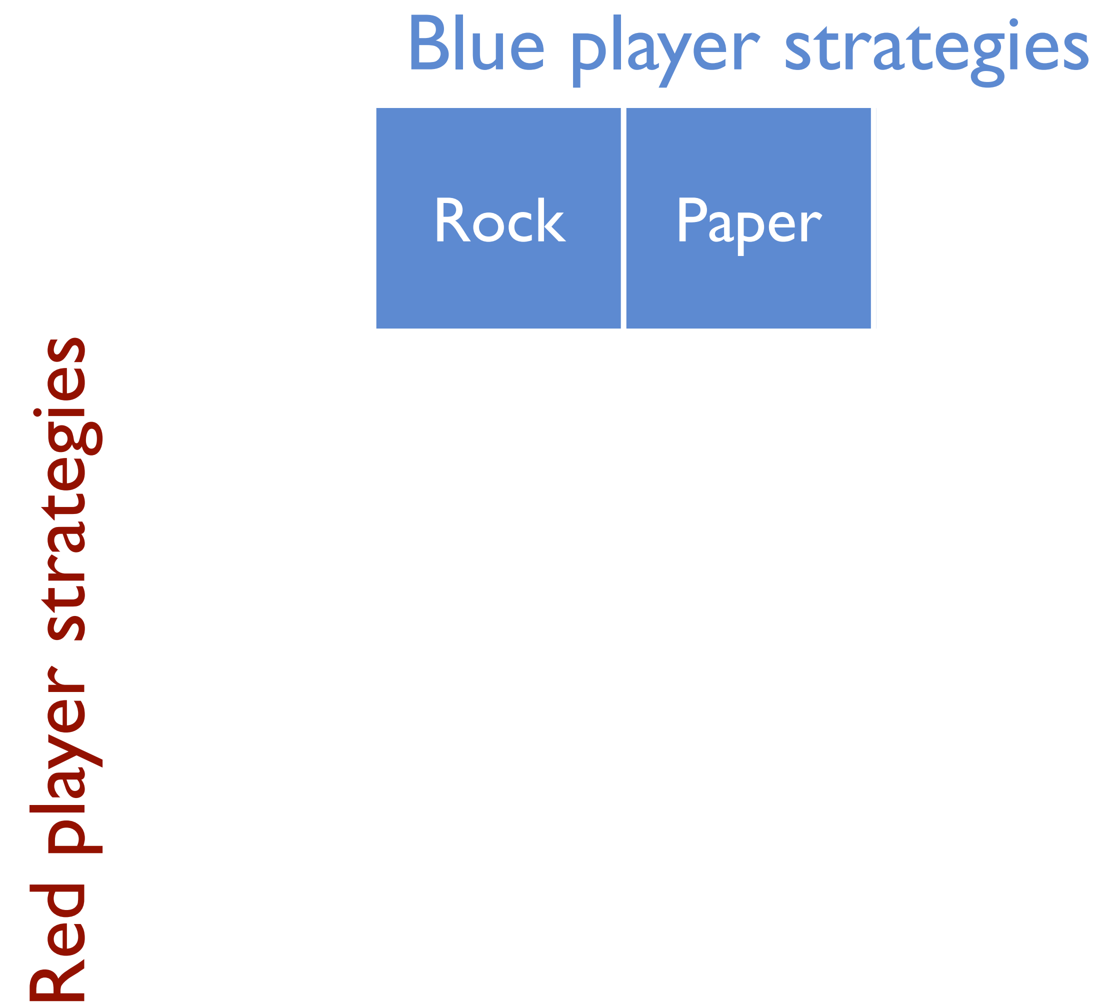

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Red player strategies

Blue player strategies

Rock	Paper	Scissors

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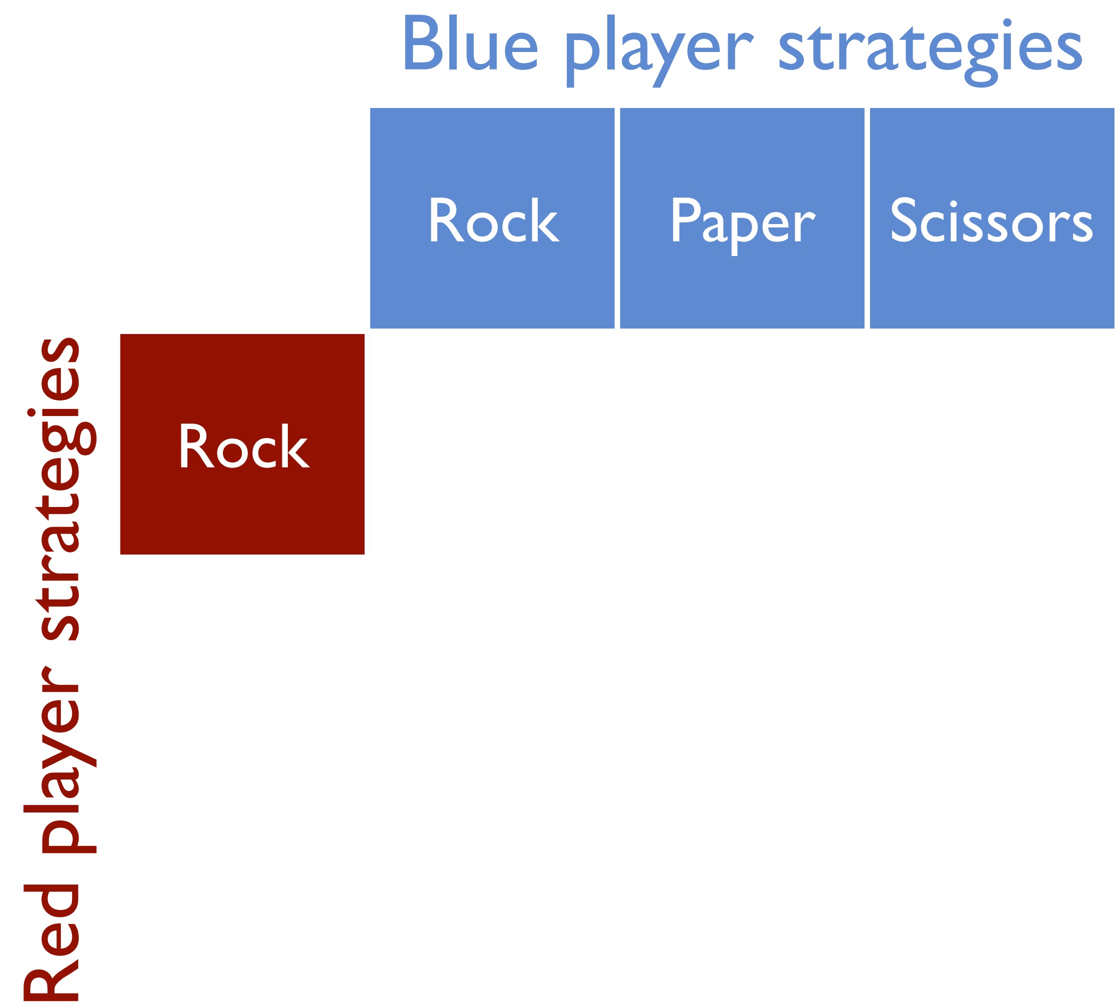

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Red player strategies

	Rock	Paper	Scissors
Rock	$0, $0		

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Paper			

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Blue player strategies (columns); Red player strategies (rows)

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Paper	$1, $0	$0, $0	$0, $1

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(Pure) Nash equilibrium

A chosen strategy for each player such that no player can improve its utility by changing its strategy

Can you find a Nash equilibrium in R-P-S?

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No pure Nash equilibrium!

Prisoner's dilemma

	Blue prisoner	
Red prisoner	Cooperate	Defect

Prisoner's dilemma

		Blue prisoner	
		Cooperate	Defect
Red prisoner	Cooperate		

Prisoner's dilemma

		Blue prisoner	
		Cooperate	Defect
Red prisoner	Cooperate	-1, -1	

Prisoner's dilemma

	Blue prisoner: Cooperate	Blue prisoner: Defect
Red prisoner: Cooperate	-1, -1	-12, 0

Prisoner's dilemma

Red prisoner	Blue prisoner: Cooperate	Blue prisoner: Defect
Cooperate	-1, -1	-12, 0
Defect		

Prisoner's dilemma

Red prisoner \ Blue prisoner	Cooperate	Defect
Cooperate	-1, -1	-12, 0
Defect	0, -12	

Prisoner's dilemma

		Blue prisoner	
		Cooperate	Defect
Red prisoner	Cooperate	-1, -1	-12, 0
	Defect	0, -12	-5, -5

Prisoner's dilemma

Red prisoner	Blue prisoner: Cooperate	Blue prisoner: Defect
Cooperate	-1, -1 →	-12, 0
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Prisoner's dilemma

Blue prisoner

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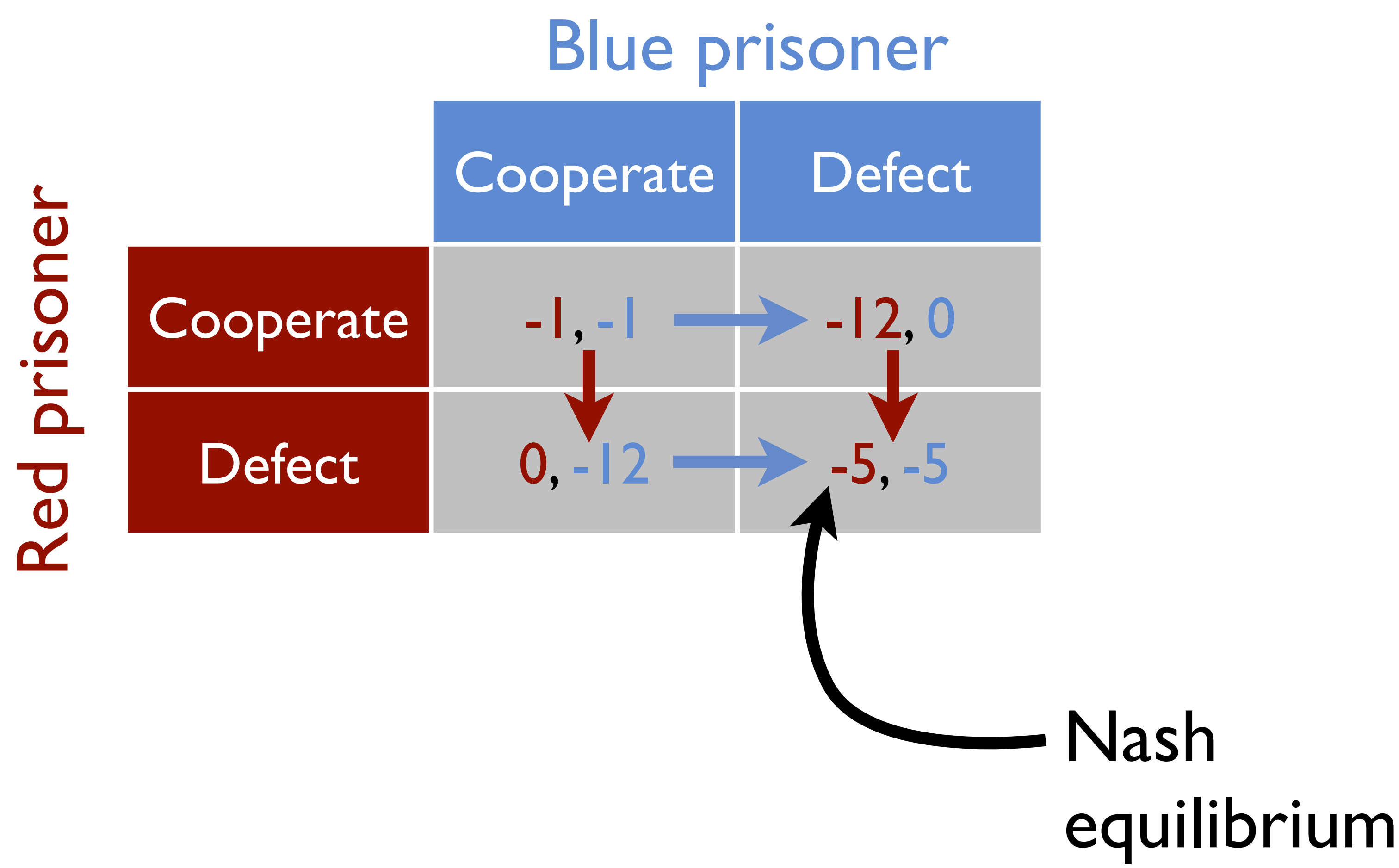

Price of Anarchy

[C. Papadimitriou, "Algorithms, games and the Internet", STOC 2001]

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Red prisoner

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Cooperate	-1, -1	-12, 0
Defect	0, -12	-5, -5

Here, PoA = 10/2 = 5.

How bad is
selfish routing
in a network?

The selfish routing game

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Given graph, latency function on each edge specifying latency as function of total flow *x* on a link

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Path latency = sum of link latencies

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Player strategy: pick a path on which to route

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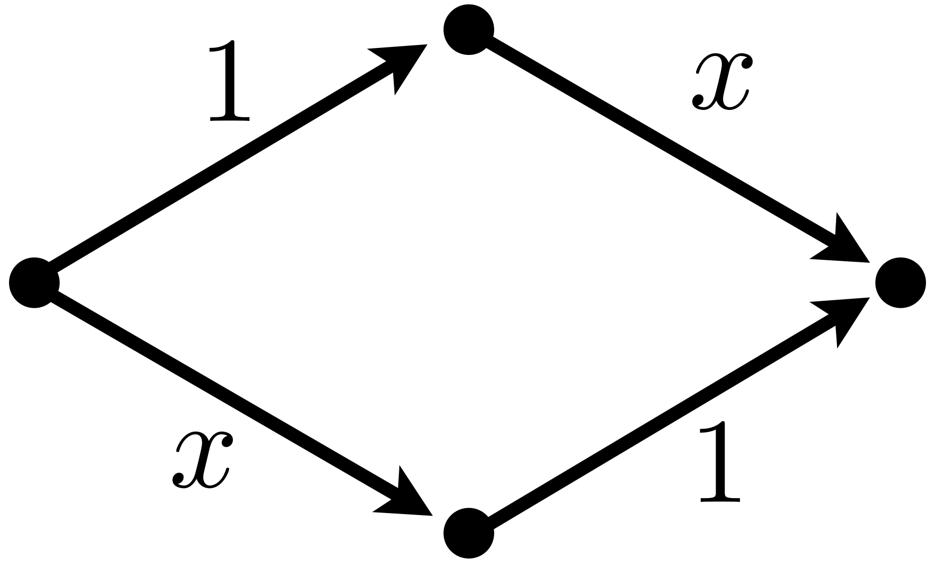
For now assume many users, each with negligible load; total 1



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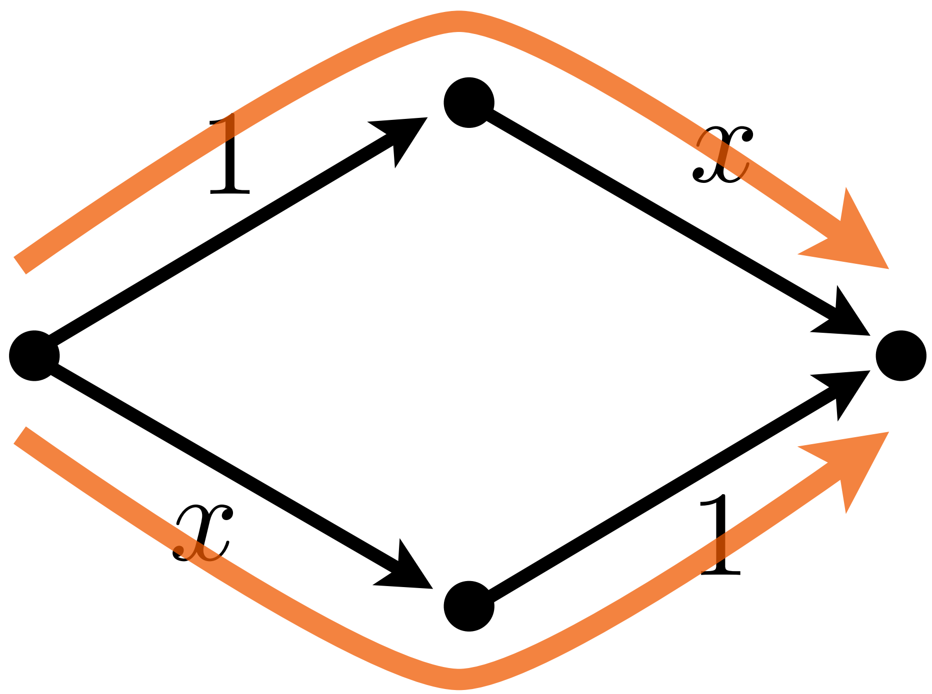

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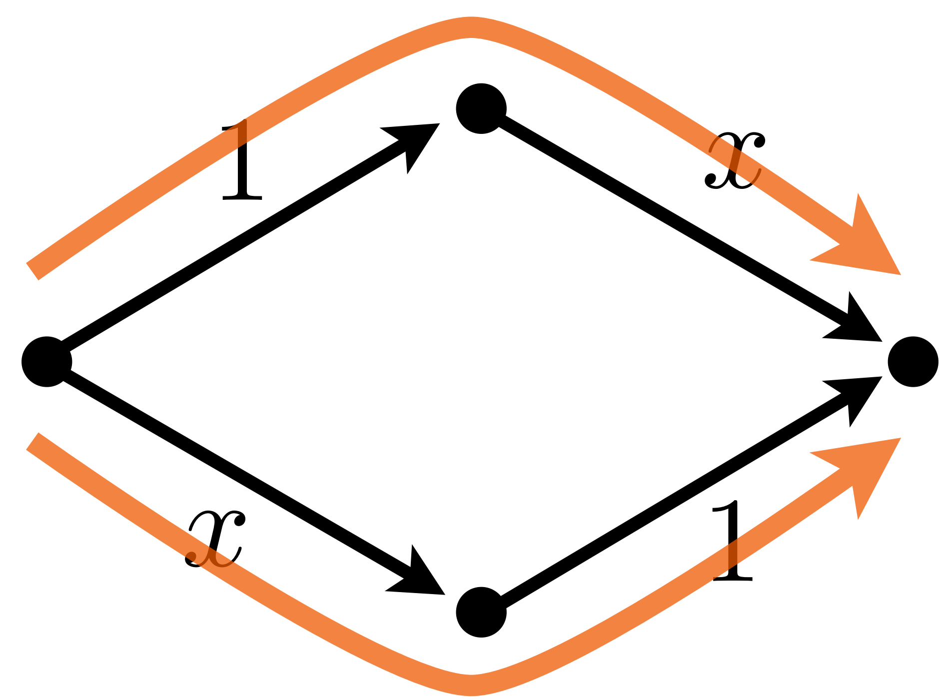

Flow *x* = 0.5 on each path;
Total latency = 1.5

Example: Braess's paradox

[Dietrich Braess, 1968]

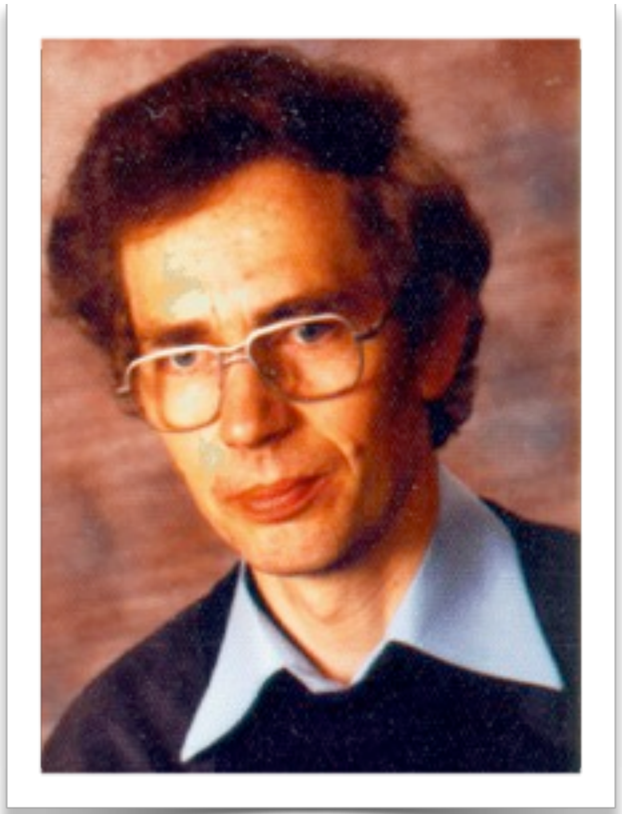

Fig 1a: D. Braess.

Example: Braess's paradox

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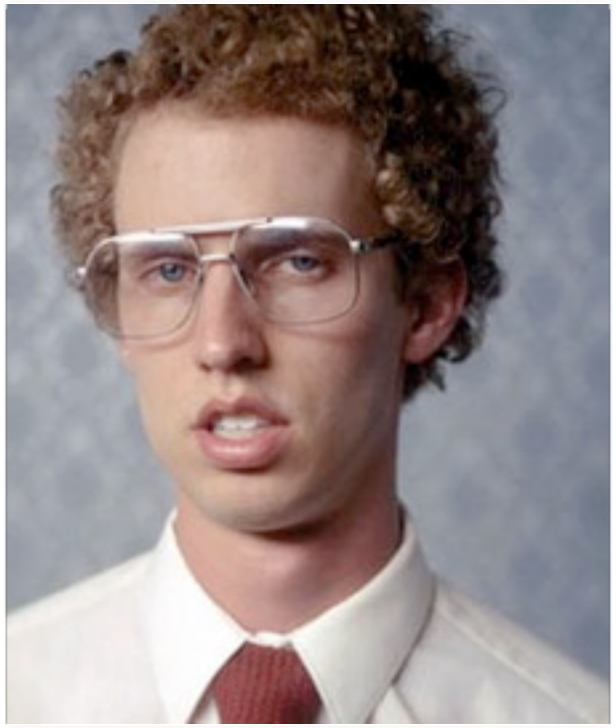

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Fig 1b: N. Dynamite.   Fig 1a: D. Braess.

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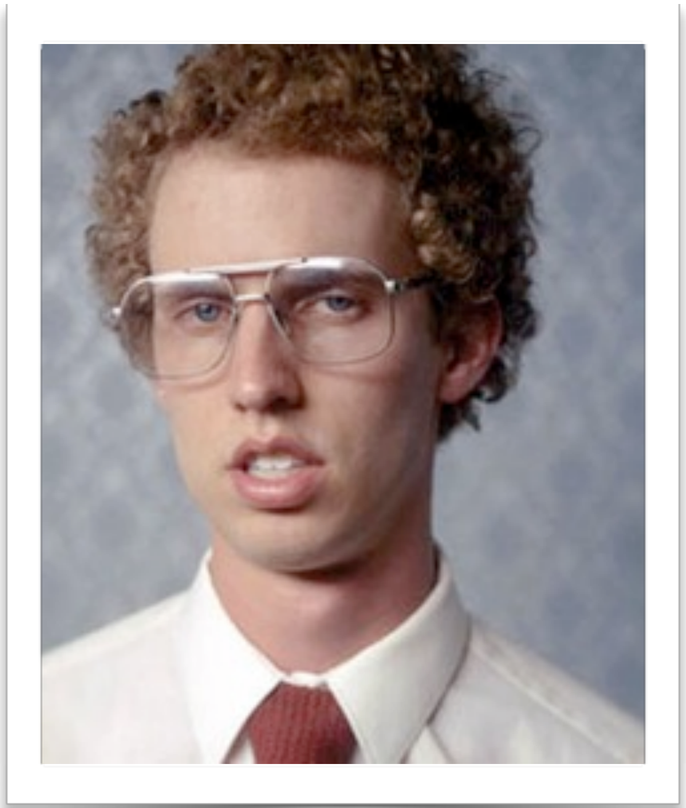

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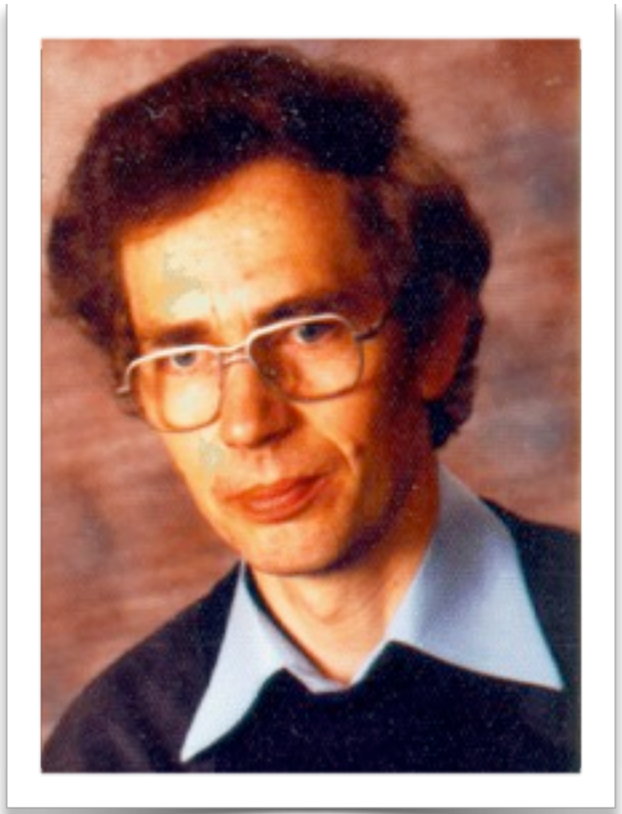

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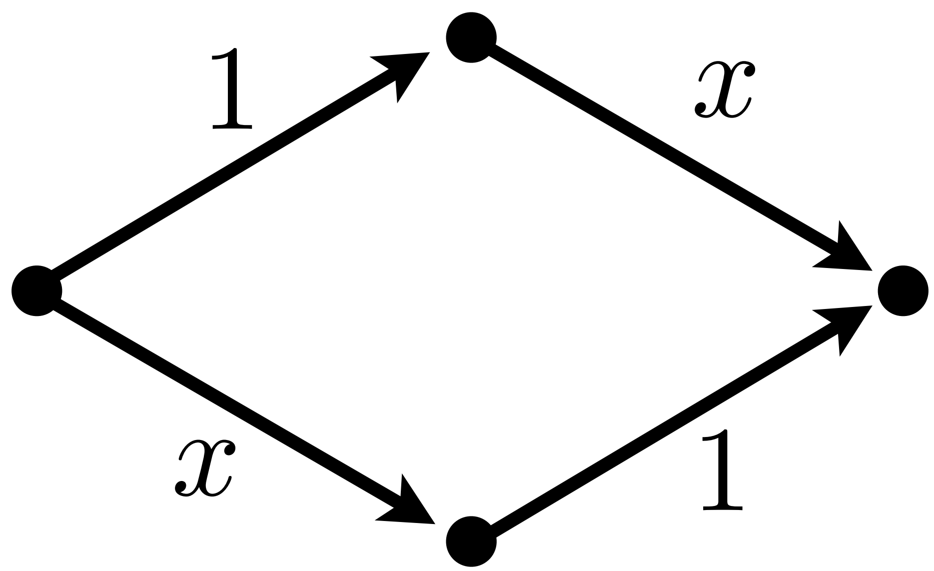

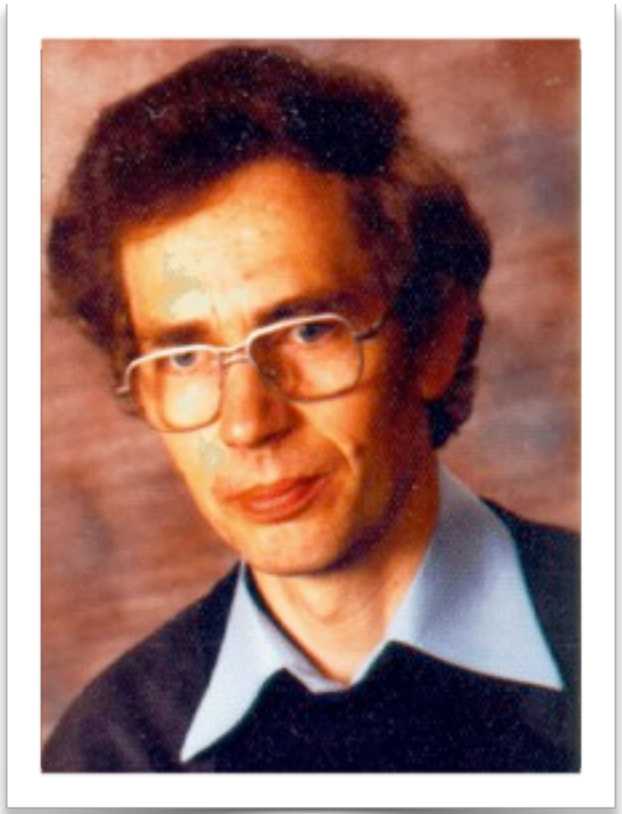

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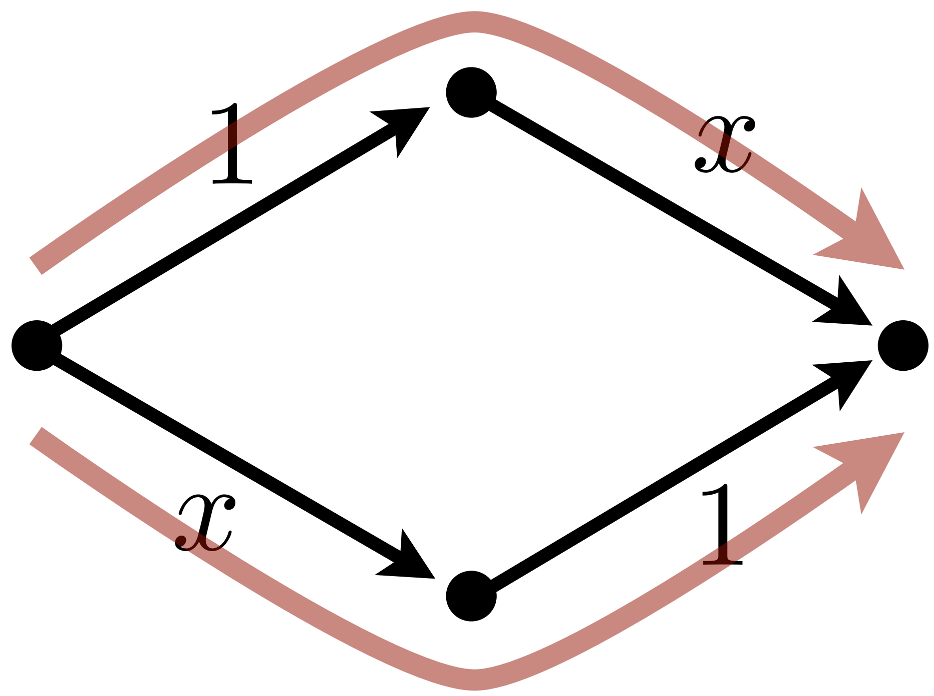

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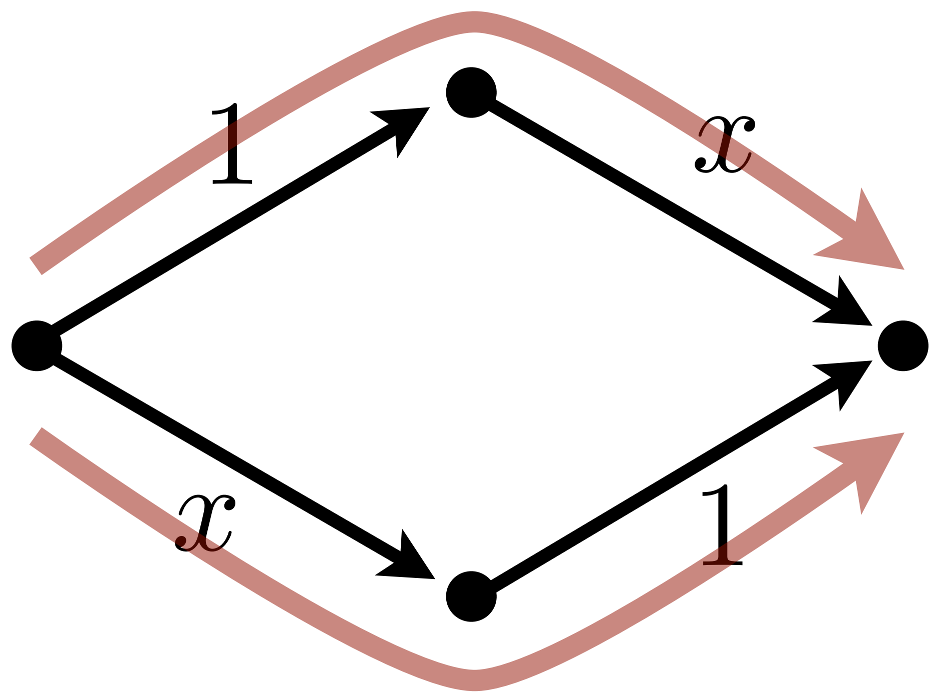

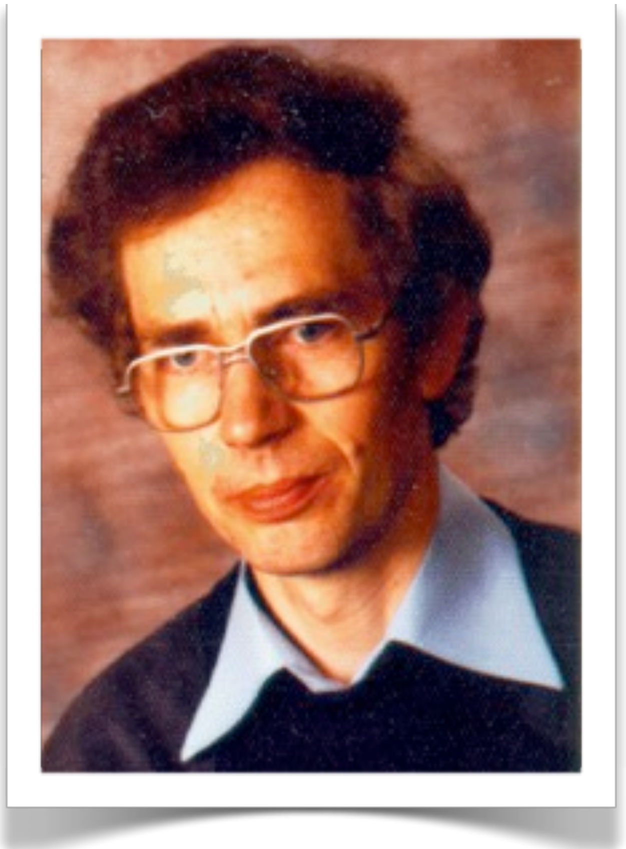

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Initially: 0.5 flow along each path; latency 1+0.5 = 1.5

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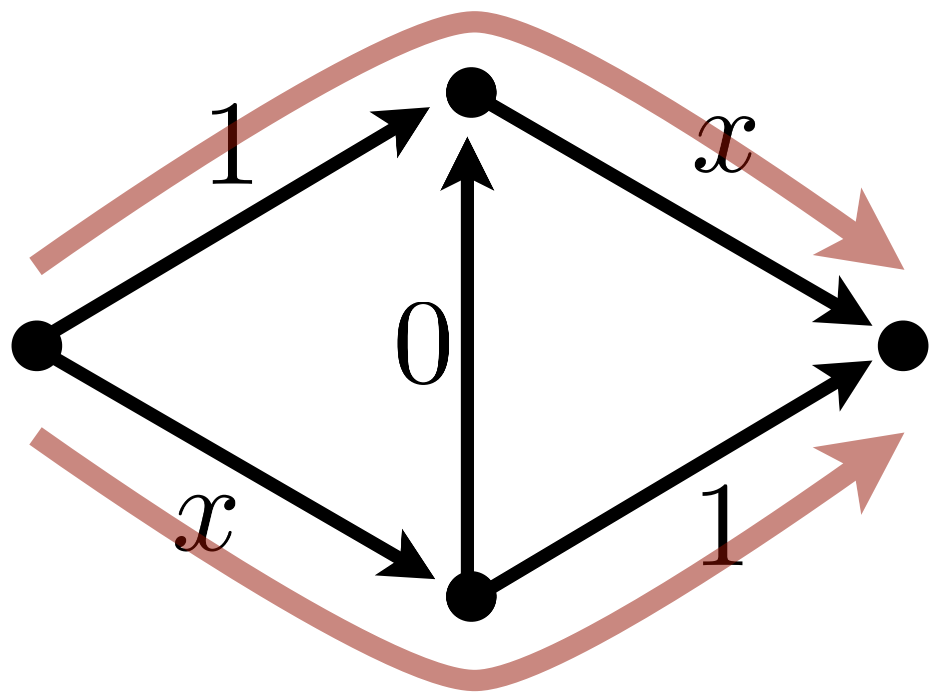

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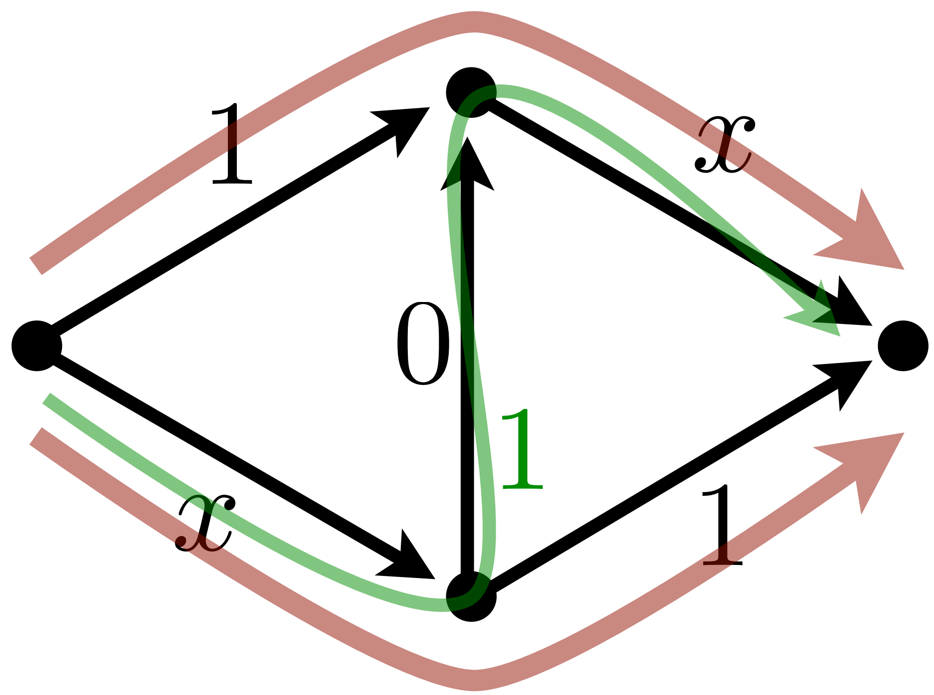

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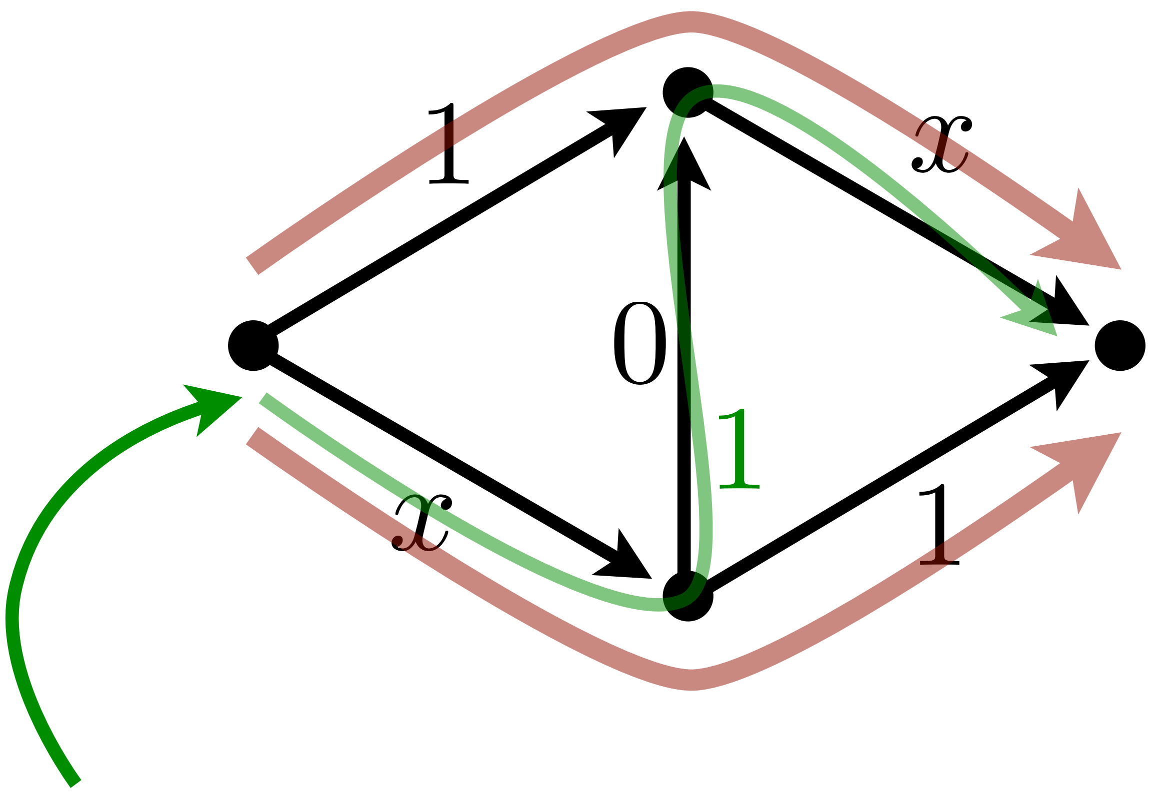
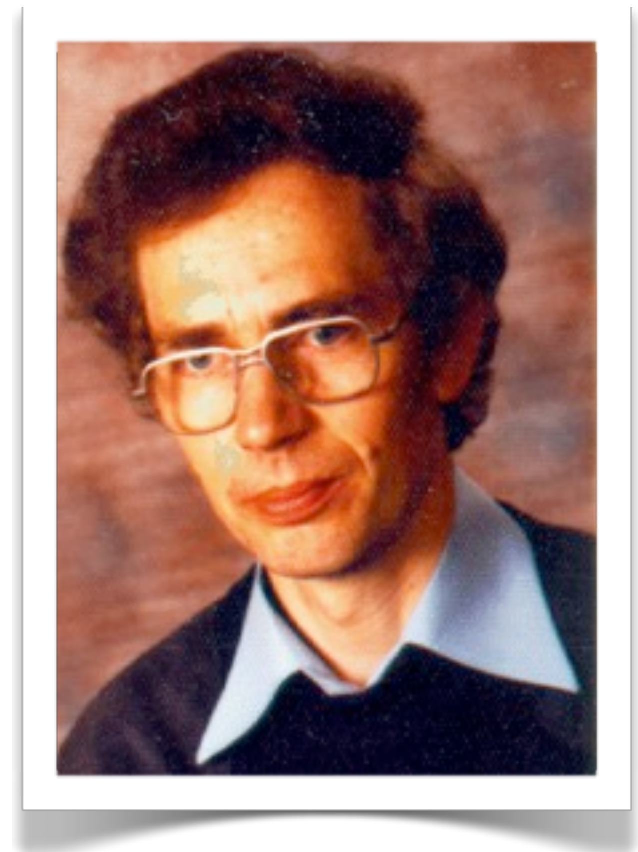


Green path is better.
Everyone switches to it!



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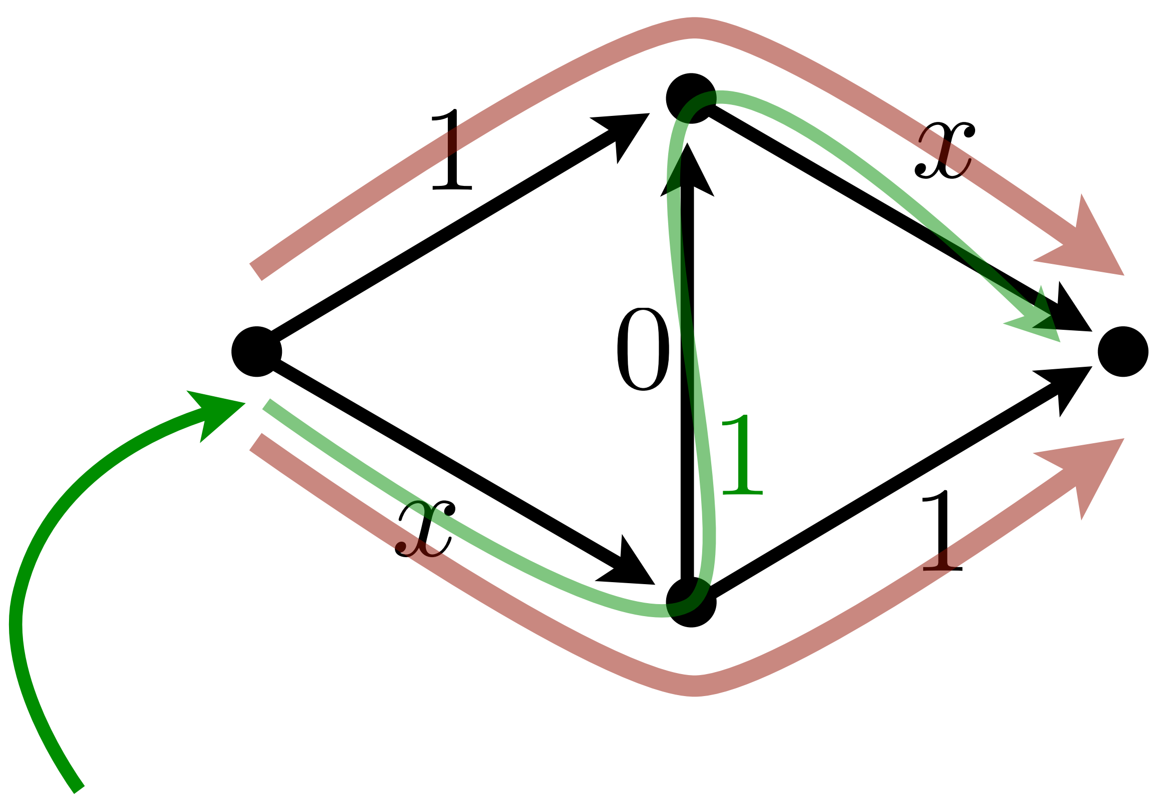

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With new link: all flow along new path; latency = 2

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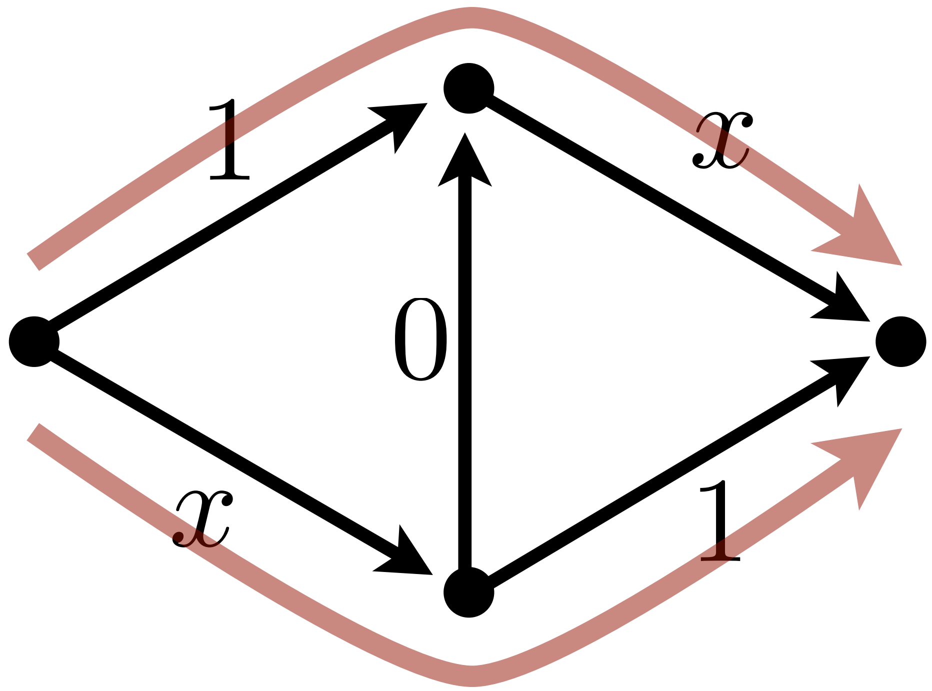

Optimal latency = 1.5

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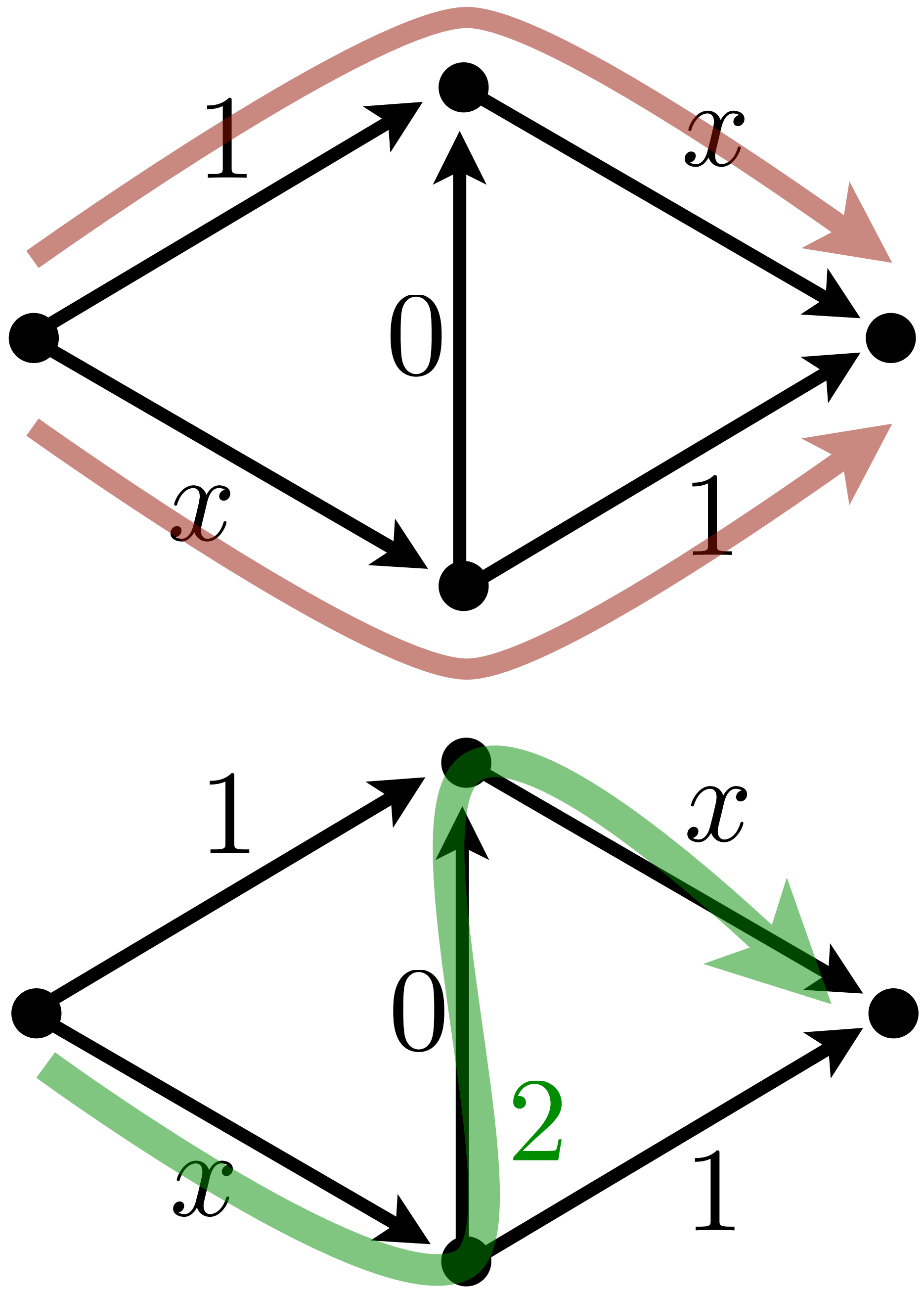

Optimal latency = 1.5

Nash equilibrium latency = 2

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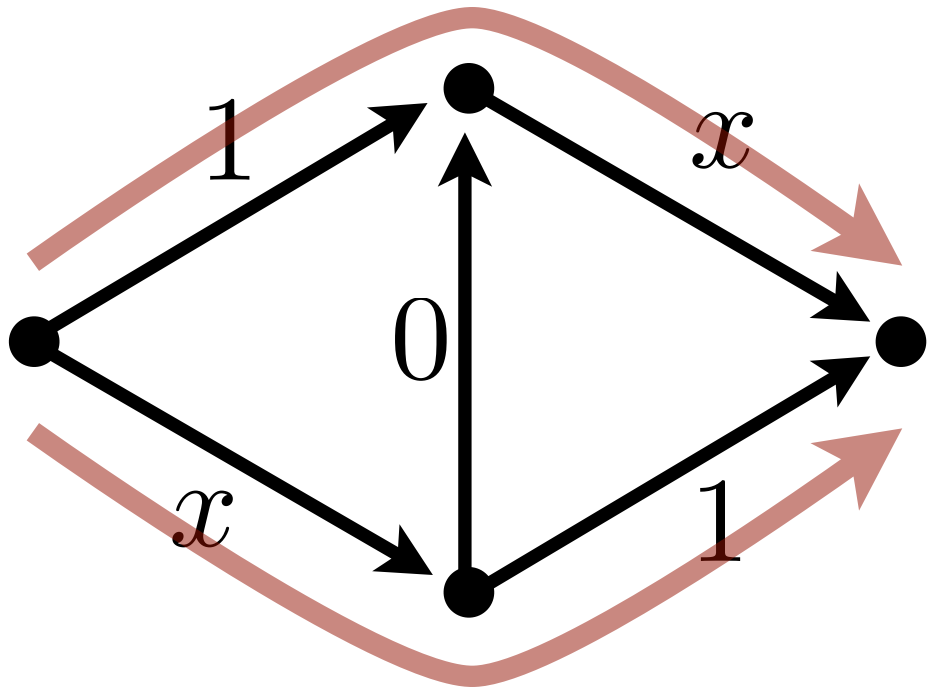

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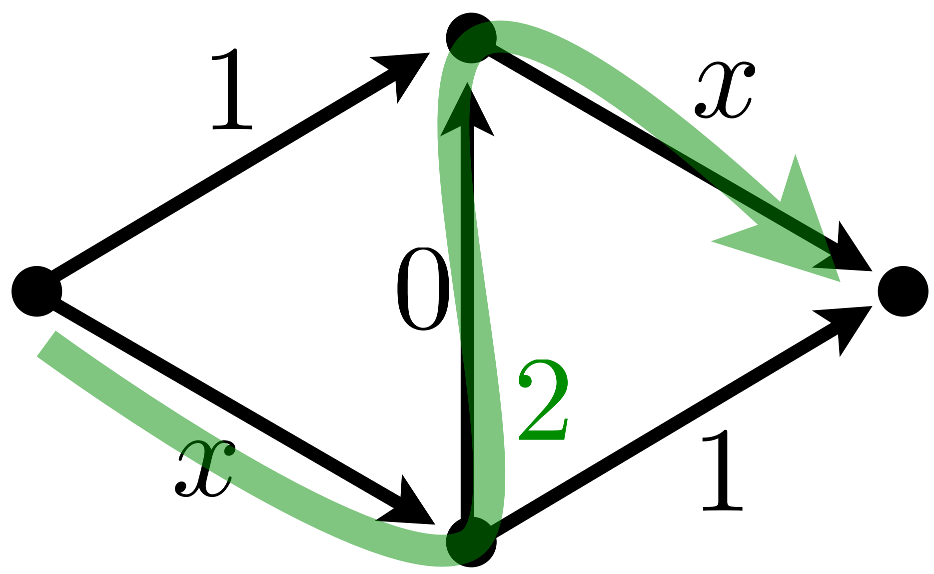
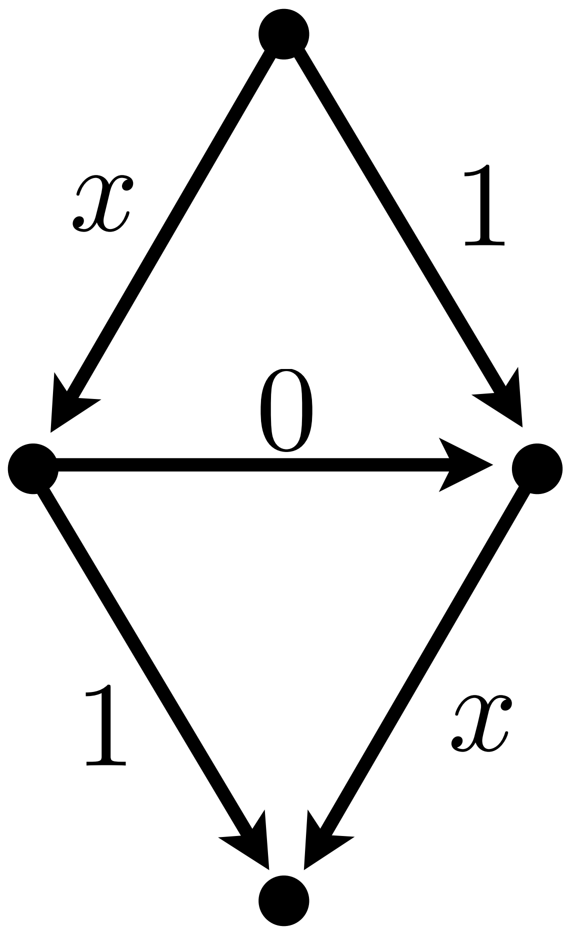


Nash equilibrium latency = 2

Thus, price of anarchy = 4/3

From links to springs



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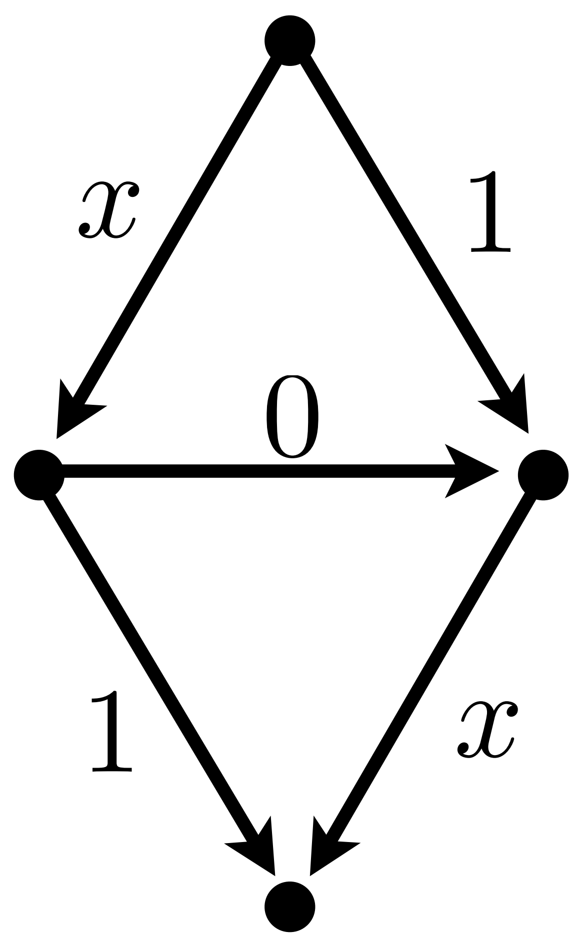

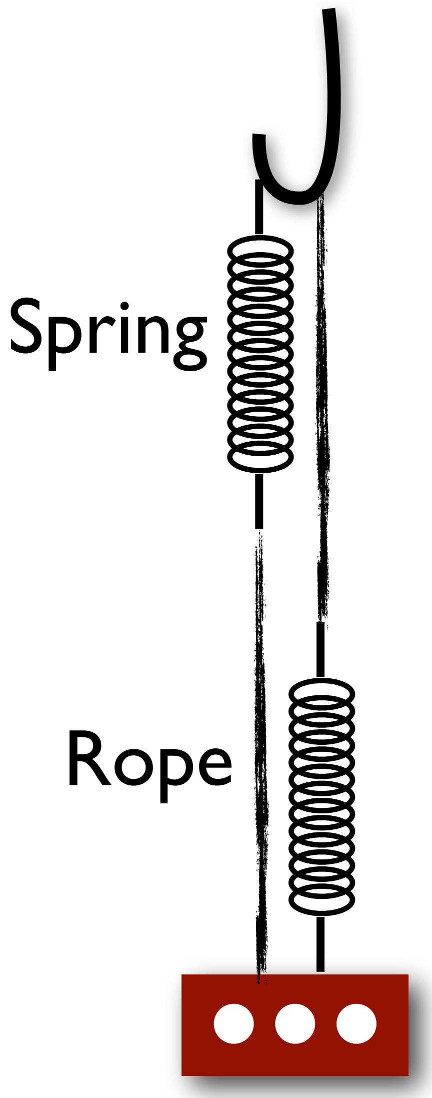

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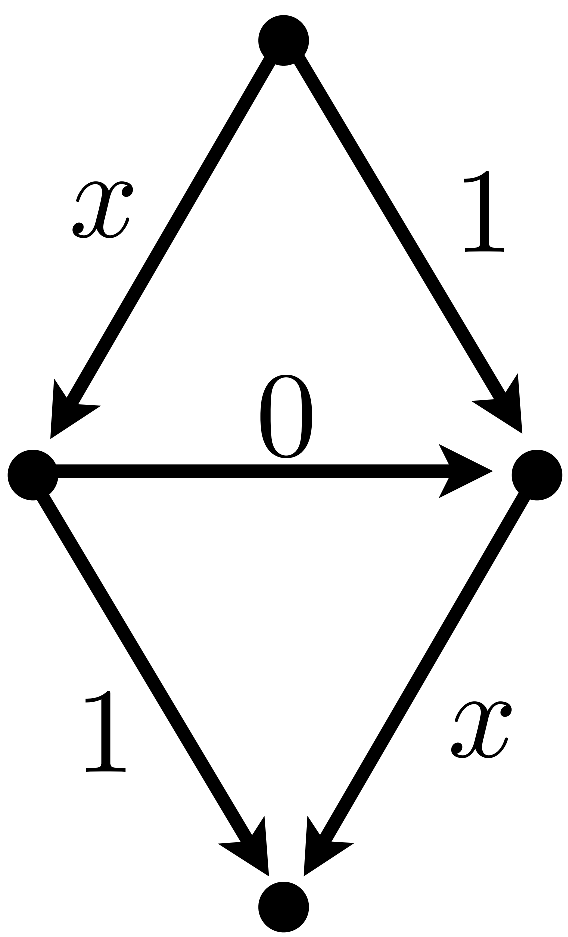

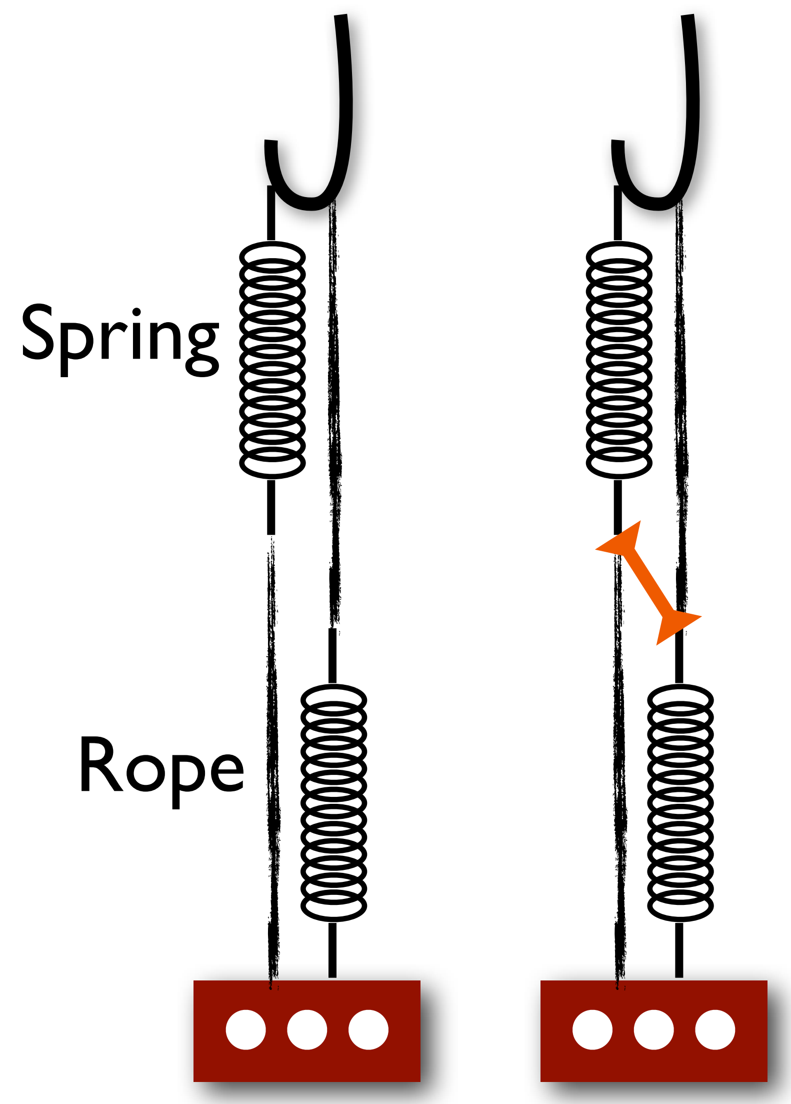

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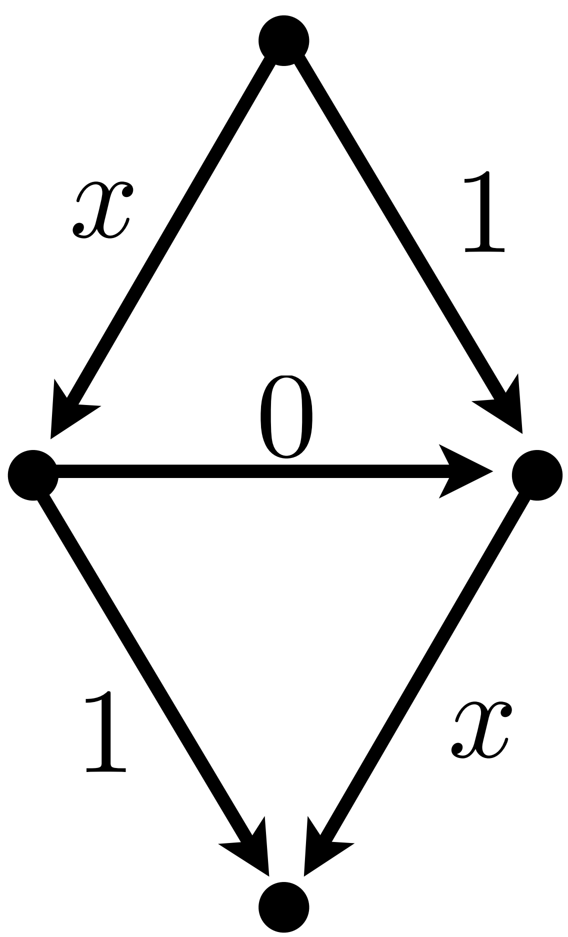

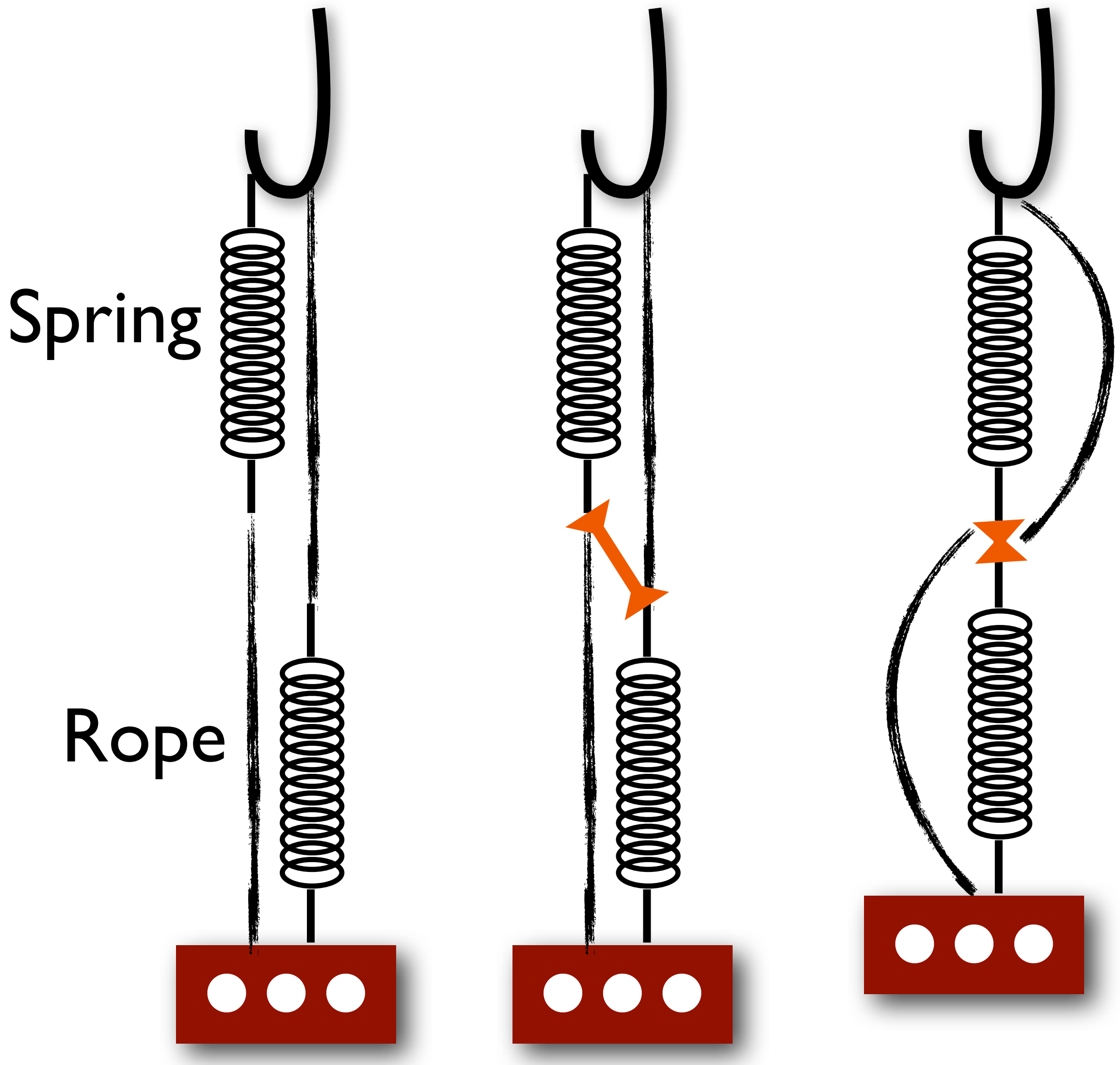

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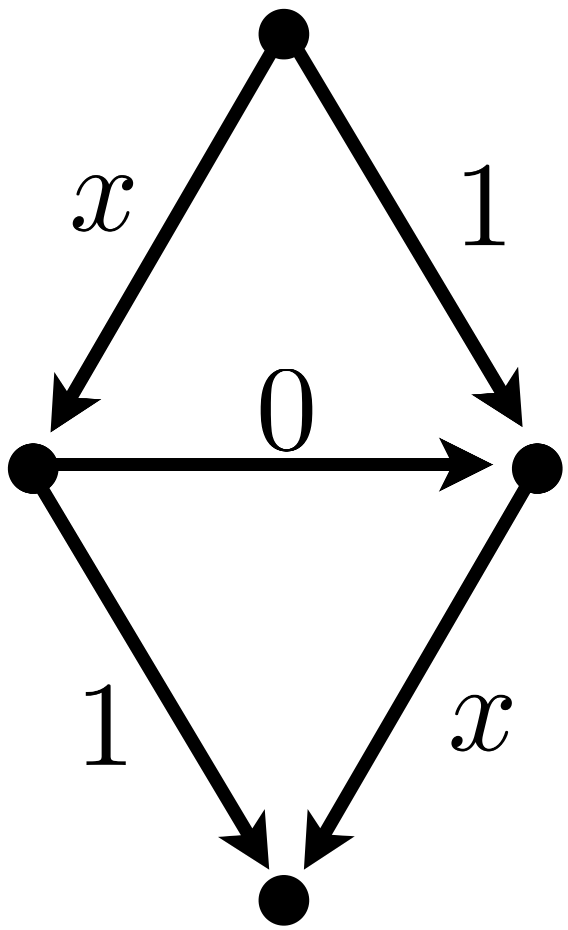
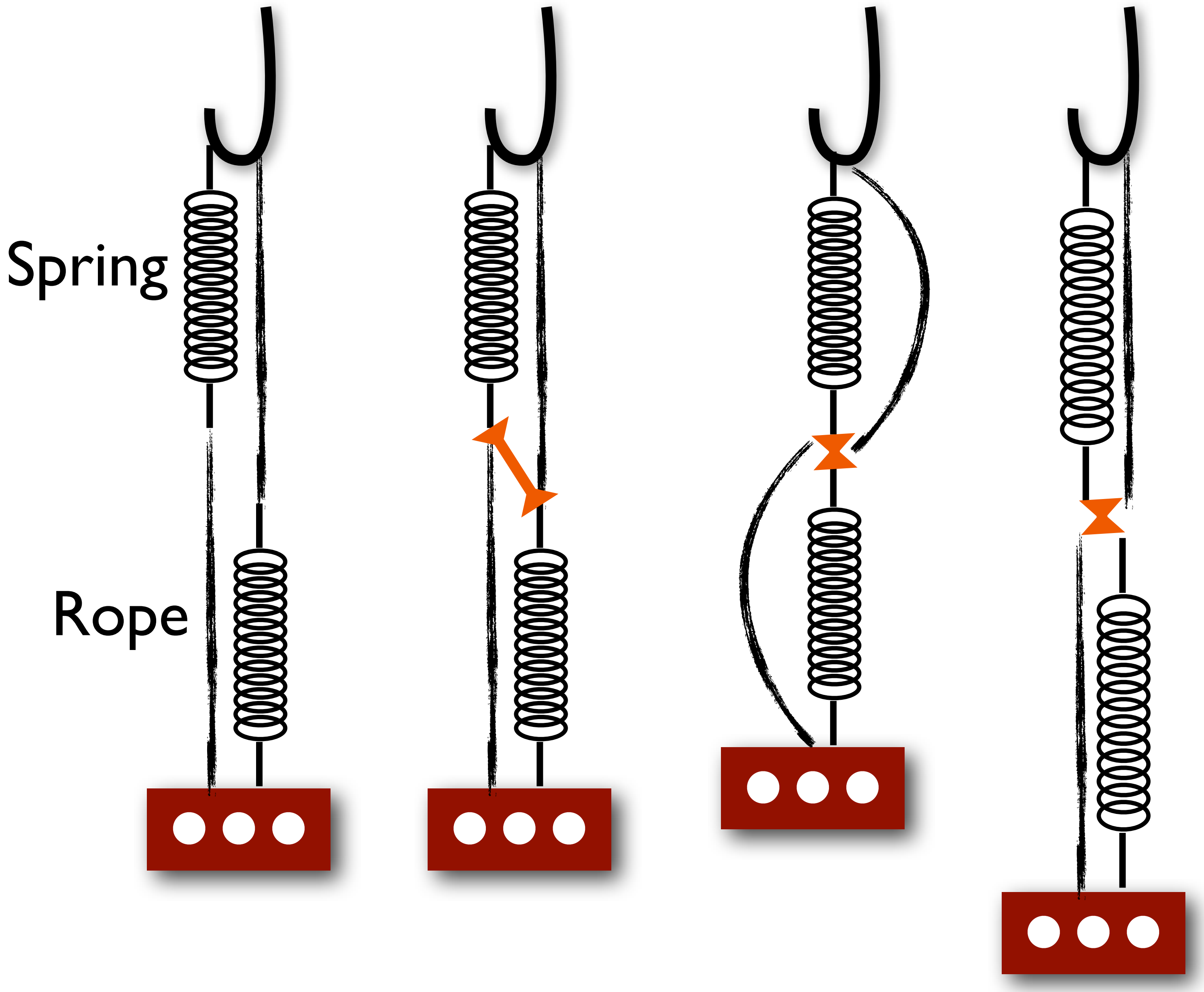
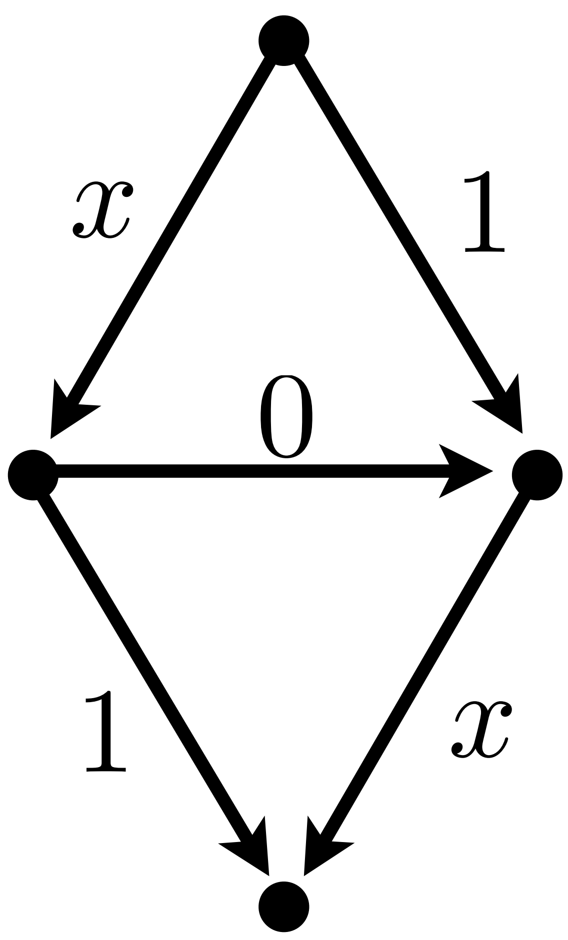
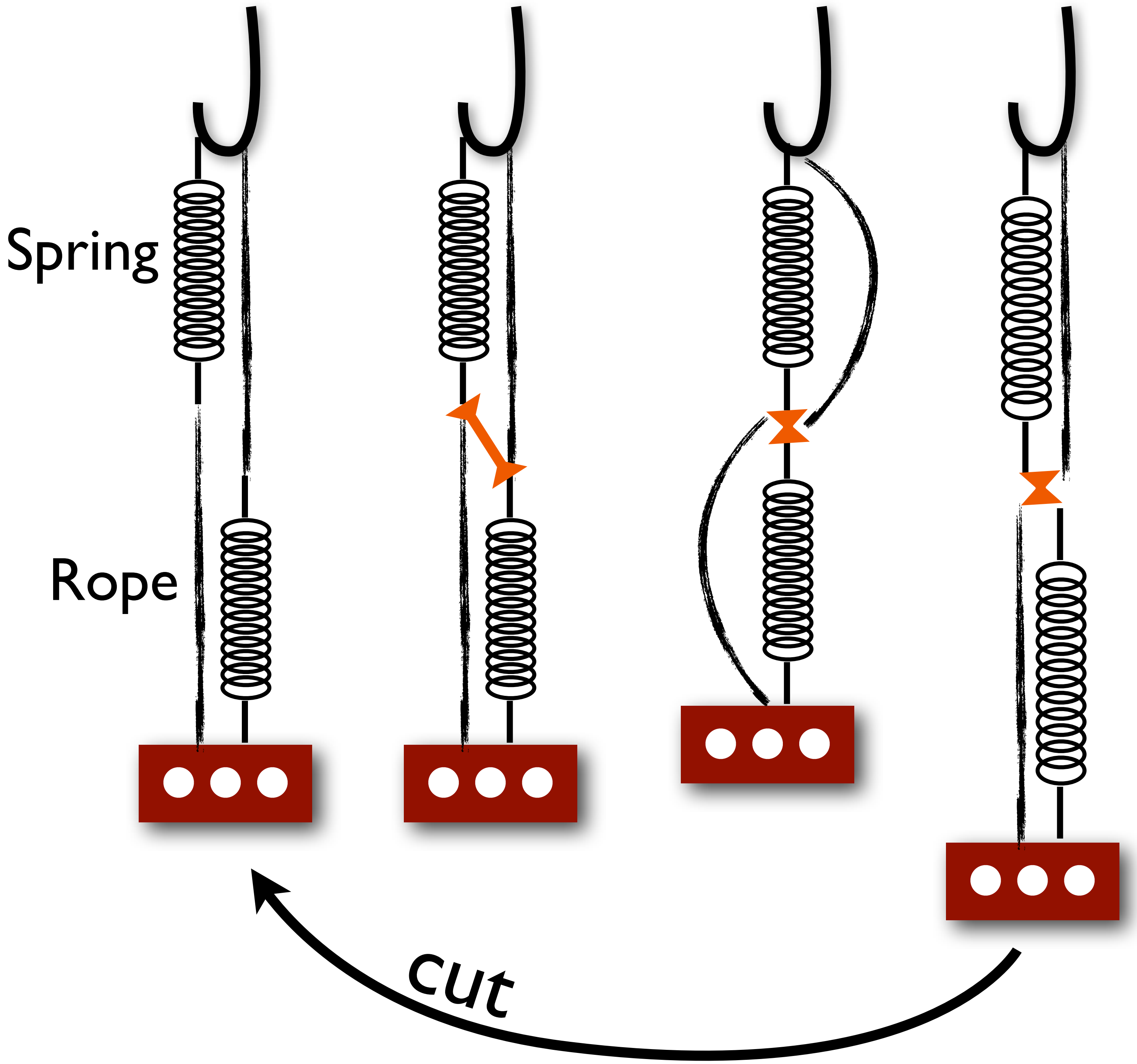




From links to springs





Going deeper

Going deeper

How bad are equilibria in real-world networks?

Going deeper

How bad are equilibria in real-world networks?

Can you design a mechanism (a game) so that selfishness is not so bad?

Going broader

Going broader

Game theory used in networking to model

Going broader

Game theory used in networking to model

- Equilibria of distributed algorithms

Going broader

Game theory used in networking to model

- Equilibria of distributed algorithms
- ISPs competing with each other

Going broader

Game theory used in networking to model

- Equilibria of distributed algorithms
- ISPs competing with each other
- Spread of new technology in social networks

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Many more applications of game theory to CS (and CS to game theory).

Going broader

Game theory used in networking to model

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- See Nisan, Roughgarden, Tardos, Vazirani's book *Algorithmic Game Theory*, available free online